**20181214数据结构作业**

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**我承诺诚实作业，没有抄袭他人！**

1. **第1题**：

**解**：由题得，这n个关键码检索成功所花费检索长度分别为1,2,3…n，因此成功检索的平均检索长度又，因此，从而，即**平均检索长度为**。

1. **第2题**：

**解**：每一步时的first、mid和last如下表所示。

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **步骤序号** | **first** | **mid** | **last** | **判断** |
| 1 | 1 | 4 | 8 | 26＜88,first = mid + 1 = 5 |
| 2 | 5 | 6 | 8 | 56＜88,first = mid + 1 = 7 |
| 3 | 7 | 7 | 8 | 找到88，结束检索 |

1. **第4题**：

**解**：以，由于题中说查找的是A[25]中每个元素，因此一定会查找成功。

1. **顺序查找**： = (1 + 2 + 3 + … 25) / 25 = **13**。
2. **二分检索**：**以每个元素的下标加圆圈表示内结点，以方块表示外结点**，给出对A[25]进行二分检索的扩充二叉树，如下所示（左子树表示检索值小于当前结点对应的值，右子树表示检索值大于当前结点对应的值）：

**L1**

12

0

1

2

5

8

**L2**

18

**L3**

15

21

**L4**

16

13

17

14

4

10

7

3

6

11

19

23

**L5**

24

22

20

9

**L6**

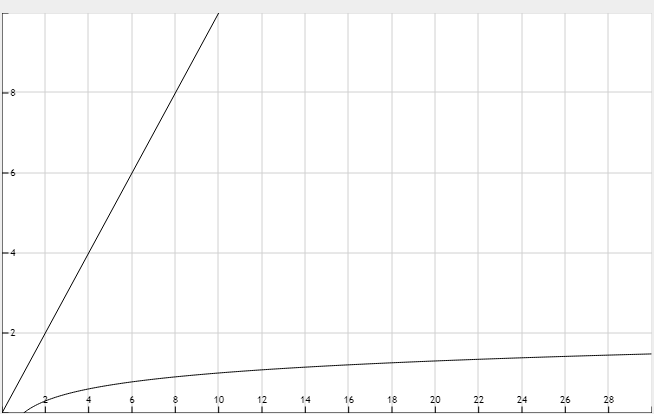
从上述扩充二叉树可以看出，**查找成功每个值所需要的查找次数**如下表所示：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** |
| **次数** | 4 | 5 | 3 | 4 | 5 | 2 | 4 | 5 | 3 | 5 | 4 | 5 | 1 | 4 | 5 | 3 | 4 | 5 | 2 | 4 | 5 | 3 | 5 | 4 | 5 |

**从而查找成功的平均检索长度(表中次数之和除以25)为**

**=** (4+5+3+4+5+…+5+4+5)/25 = 99/25 = **3.96**。

1. **分块**：由两阶段的顺序检索组成，因此查找成功的平均检索长度为 **=** (5+1)/2+(5+1)/2 = **6**。
2. **第5题**：**请查看CLion程序**

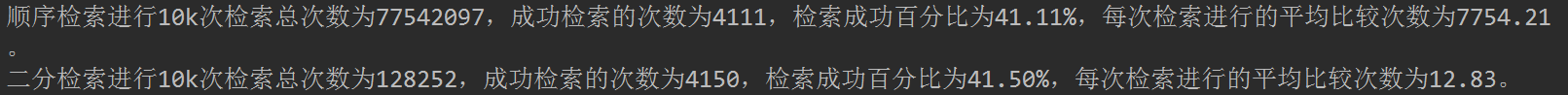
**第5小问**：

**O (n)**

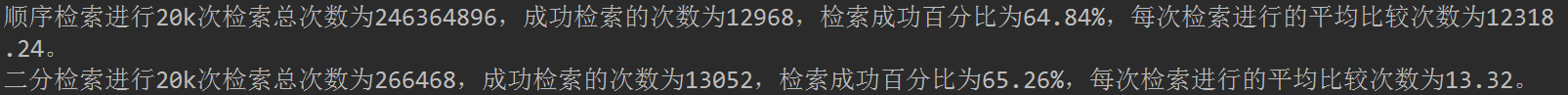
**O (log n)**

1. **如图所示(注意x轴的值较大，y轴值较小)，顺序检索是O(n)复杂度，二分检索是O(log n)复杂度，上图是算法分析得到的函数图像；**
2. **实际运行的情况，先给出几组值**：

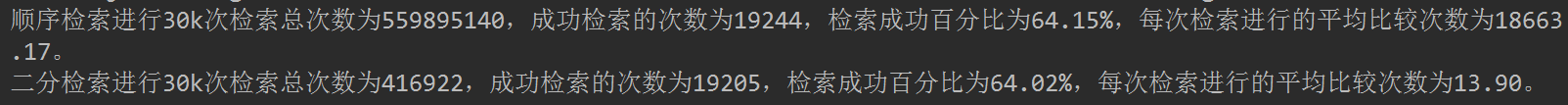
10K，值域[1-20k]数据



20K，值域[1-20k]数据



30k，值域[1-30k]数据



有如下表：

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **算法** | **总检索次数** | **成功检索次数** | **成功百分比** | **平均比较次数** |
| **顺序检索** | 77,542,097 | 4,111 | 41.11% | 7754.21 |
| **顺序检索** | 246,364,896 | 12,968 | 64.84% | 12318.24 |
| **顺序检索** | 559,895,140 | 19,244 | 64.15% | 18663.17 |
| **二分检索** | 128,252 | 4,150 | 41.50% | 12.83 |
| **二分检索** | 266,468 | 13,052 | 65.26% | 13.32 |
| **二分检索** | 416,922 | 19,205 | 64.02% | 13.90 |

可以看到，**二分检索的平均比较次数非常稳定，增长速度很缓慢，这是O(log n)算法的特征，即n的规模必须跨越很大倍的提升，log n才会有较明显的提升；而顺序检索随着n的增大，平均比较次数直线上升，是O(n)算法的典型表现**。

1. **第6题**：**请查看CLion程序**
2. **第11题**：
3. **首先求出每个元素对23取余的余数，如下表所示**：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **32** | **75** | **29** | **63** | **48** | **94** | **25** | **46** | **18** | **70** |
| **余数** | 9 | 6 | 6 | 17 | 2 | 2 | 2 | 0 | 18 | 1 |

1. **插入元素32，得到如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  |  |  |  |  |  |  |  | 32 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. **插入元素75，得到如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  |  |  |  |  | 75 |  |  | 32 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. **插入元素29，发生冲突，使用线性探测解决冲突得元素29插入在下标7，得如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  |  |  |  |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. **插入元素63，得如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  |  |  |  |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  | 63 |  |  |  |  |  |  |

1. **插入元素48，得如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  | 48 |  |  |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  | 63 |  |  |  |  |  |  |

1. **插入元素94，发生冲突，使用线性探测解决得元素94插入在下标3，得如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  | 48 | 94 |  |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  | 63 |  |  |  |  |  |  |

1. **插入元素25，发生冲突，使用线性探测解决得元素25插入在下标4，得如下散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** |  |  | 48 | 94 | 25 |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  | 63 |  |  |  |  |  |  |

1. **插入元素46，18，70得如下最终散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** |
| **元素** | 46 | 70 | 48 | 94 | 25 |  | 75 | 29 |  | 32 |  |  |  |  |  |  |  | 63 | 18 |  |  |  |  |  |

1. **根据上述步骤，我们可以得到每个元素查找成功的次数如下表**：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **32** | **75** | **29** | **63** | **48** | **94** | **25** | **46** | **18** | **70** |
| **查找成功次数** | 1 | 1 | 2 | 1 | 1 | 2 | 3 | 1 | 1 | 1 |

\*\*因此**平均查找长度**为(1+1+2+1+1+2+3+1+1+1)/10 = **1.4**。

1. **第13题**：

**首先用每个元素对13取余，得到如下表**：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **12** | **23** | **45** | **57** | **20** | **03** | **78** | **31** | **15** | **36** |
| **余数** | 12 | 10 | 6 | 5 | 7 | 3 | 0 | 5 | 2 | 10 |

1. **利用线性探查法解决冲突，有如下步骤**：
2. 插入元素**12,23,45,57,20,03,78**得如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 |  |  | 03 |  | 57 | 45 | 20 |  |  | 23 |  | 12 |

1. 插入元素**31**，发生冲突，使用线性探查解决得**元素31插入在下标6**，得如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 |  |  | 45 | 03 | 57 | 31 | 20 |  |  | 23 |  | 12 |

1. 插入元素**15**，得如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 |  | 15 | 45 | 03 | 57 | 31 | 20 |  |  | 23 |  | 12 |

1. 插入元素**36**，发生冲突，使用线性探查解决得**元素36插入在下标11**，得如下**最终散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 |  | 15 | 45 | 03 | 57 | 31 | 20 |  |  | 23 | 36 | 12 |

1. 根据上述过程，得到每个元素的查找成功的检索长度，则得到如下查找次数表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **12** | **23** | **45** | **57** | **20** | **03** | **78** | **31** | **15** | **36** |
| **查找成功次数** | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 2 |

则**查找成功的平均检索长度为(1+1+1+1+1+2+1+2+1+2)/10 = 1.4。**

1. **查找不成功是指对每个散列下标来讲，使用线性探查向后探寻，直至遇到第一个为空的元素停止查找，**因此对每个散列下标查找不成功次数如下表所示：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **查找失败长度** | 2 | 1 | 3 | 2 | 1 | 5 | 4 | 3 | 2 | 1 | 5 | 4 | 3 |

则**查找不成功的平均检索长度为(2+1+3+2+1+5+4+3+2+1+5+4+3)/13 = 36/13**。

1. **使用再散列法寻找下一个下标**：
2. 首先插入元素**12,23,45,57,20,03,78**得如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 |  |  | 03 |  | 57 | 45 | 20 |  |  | 23 |  | 12 |

1. 插入元素**31**，发生冲突，使用再散列法计算插入下标：=5，rh(31)=(7\*31)%11+1=9，=(5+9)%13=1，发现下标1没有元素，因此可以插入得到如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 | 31 |  | 03 |  | 57 | 45 | 20 |  |  | 23 |  | 12 |

1. 插入元素15，得到如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 | 31 | 15 | 03 |  | 57 | 45 | 20 |  |  | 23 |  | 12 |

1. 插入元素36，发生冲突，使用再散列法计算插入下标：=10，rh(36)=(7\*36)%11+1=11，=(10+11)%13=8，发现下标8没有元素，因此可以插入得到如下**最终散列表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| **元素** | 78 | 31 | 15 | 03 |  | 57 | 45 | 20 | 36 |  | 23 |  | 12 |

1. 根据上述过程，每个元素查找成功的检索长度如下表所示：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **12** | **23** | **45** | **57** | **20** | **03** | **78** | **31** | **15** | **36** |
| **余数** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 |

则**检索成功的平均检索长度为(1\*8+2\*2)/10 = 1.2**。

1. **第16题**：
2. **解**：由于总共有12个关键码且负载因子为0.6，因此M=12/0.6=20，则除余法用于取余的因子应当取小于20的最大素数即19，因此散列函数为**h(K) = K % 19**。
3. **首先1)中散列函数求出每个元素对应的余数，如下表**：

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **元素** | **26** | **25** | **20** | **33** | **21** | **24** | **45** | **204** | **42** | **38** | **29** | **31** |
| **余数** | 7 | 6 | 1 | 14 | 2 | 5 | 7 | 14 | 4 | 0 | 10 | 12 |

根据书上所讲，**选取小于19的最大素数为17得到再散列函数rh(K) = k%17 + 1**，从而下一个开放地址的**计算公式为**。

\*\*\*\*带有 标识的是新插入表中的元素，发生冲突的地方使用深红色加粗标识\*\*\*\*

1. 首先，插入元素**26**,**25**,**20**,**33**,**21**,**24**，得如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** |  | 20 | 21 |  |  | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** |  |  |  |  | 33 |  |  |  |  |  |

1. 插入元素**45**，**发生冲突**，根据再散列函数计算得rh(45)=45%17+1=12，则再次计算得到的下标=(7+12)%19=0，发现下标0没有元素，插入得到如下表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  |  | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** |  |  |  |  | 33 |  |  |  |  |  |

1. 插入元素**204**，**发生冲突**，根据再散列函数计算得rh(204) = 204%17+1 = 1，则再次计算得到的下标 = (1+14)%19 = 15，发现下标15没有元素，插入得到如下表：

­­

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  |  | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** |  |  |  |  | 33 | 204 |  |  |  |  |

1. 插入元素**42**，得到如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  | 42 | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** |  |  |  |  | 33 | 204 |  |  |  |  |

1. 插入元素**38**，**发生冲突**，根据再散列函数计算得rh(38) = 38%17+1 = 5，则再次计算得到的下标 = (0+5)%19 = 5，**发生冲突**，继续计算=(0+5\*2)%19=10，发现下标10处没有元素，插入得到如下表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  | 42 | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** | 38 |  |  |  | 33 | 204 |  |  |  |  |

1. 插入元素**29**，**发生冲突**，根据再散列函数计算得rh(29) = 29%17+1 = 13，则再次计算得到的下标 = (10+13)%19 = 4，**发生冲突**，继续计算=(10+13\*2)%19=17，发现下标17处没有元素，插入得到如下散列表：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  | 42 | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** | 38 |  |  |  | 33 | 204 |  | 29 |  |  |

1. 插入元素**31**，得到如下**最终散列表**：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **­下标** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **元素** | 45 | 20 | 21 |  | 42 | 24 | 25 | 26 |  |  |
| **­下标** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** |  |
| **元素** | 38 |  | 31 |  | 33 | 204 |  | 29 |  |  |

**因此，上表即为最终的经双散列法解决冲突的散列函数表，共发生6次冲突。**